NOTATION

x, y, z, Cartesian coordinates; L, characteristic spatial scale of temperature variations; v, mean propagation velocity of thermal phonons; τ_p, l_p, relaxation time and mean free path of a thermal phonon; τ, characteristic time of temperature variations; a, thermal diffusivity; λ, thermal conductivity; q, heat flux power; T_0, initial temperature of the surface of the crystal; K_0, MacDonald function; T, absolute temperature; ΔH, reaction heat; ρ, density of the material; C, heat capacity; c_{ij}, elastic coefficients.

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SOME PROPERTIES OF THE HEAT-TRANSFER PROCESS IN A MOTIONLESS MEDIUM, TAKING ACCOUNT OF HEAT-FLUX RELAXATION

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The solution of a hyperbolic system of heat-transfer equations in which account is taken of the temperature dependence of the thermal conductivity and relaxation time of the heat flux is investigated.

1°. A wide range of physical problems leads to the need for detailed study of heat transfer. The Fourier law is most often used to describe this process

$$\mathbf{W} = \mathbf{W}_F = -\varkappa \operatorname{grad} T. \tag{1}$$

However, the limits of applicability of the Fourier law are prescribed by the requirement of smallness of the free-path length and time of the particles in comparison with the characteristic space–time scales of temperature variation and are often overstepped in the case of intense heat transfer. Note also that the heat flux cannot exceed the maximum value determined by the conventional situation in which all the particles suddenly change their direction of motion and move in the same direction.

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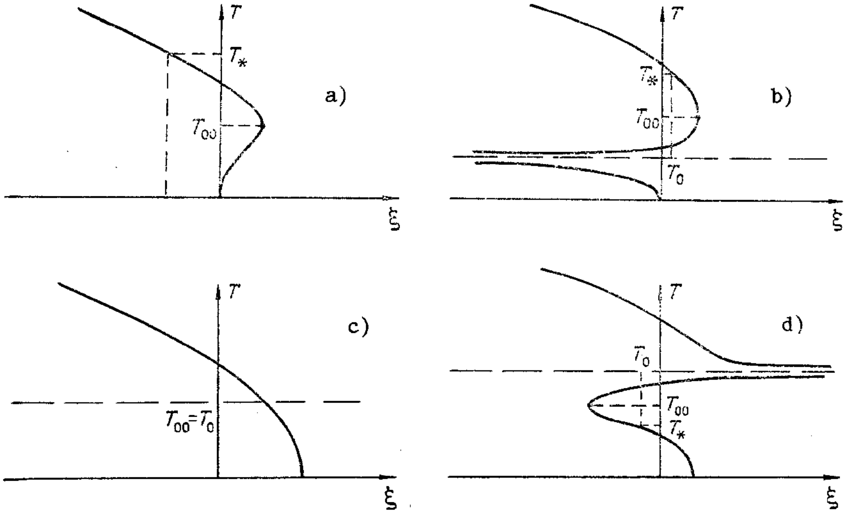

Fig. 1. Qualitative character of integral curves in Eq. (17): a) $T_0 = 0$ ($c_1 = 0$); b) $0 < T_0 < T_{00}$ ($0 < c_1 < D$); c) $T_0 = T_{00}$ ($c_1 = D$); d) $T_0 > T_{00}$ ($c_1 > D$).

This upper bound on the heat flux is of fundamental importance in describing the electron heat conduction of hot plasma [1, 2]. However, the method of taking account of heat-flux constraints most widespread in numerical calculations has definite shortcomings, as shown in [2]. Strictly speaking, the kinetic equations should be considered. However, in view of the considerable demand on machine time, the use of the kinetic equations is not always appropriate for problems complicated by taking account of many nonlinear effects.

On the basis of the foregoing, the urgency of discovering and investigating other physically meaningful mathematical models for heat transfer may be understood.

2°. One such model, known since Maxwell's work [3], is based on the following equation for the heat flux

$$\tau \frac{\partial \mathbf{W}}{\partial t} + \mathbf{W} = - \varkappa \operatorname{grad} T, \tag{2}$$

where τ is the relaxation time of the heat flux, equal to the free-path time of the particle in order of magnitude.

In the simplest case, when the thermal conductivity $\varkappa$ and relaxation time of the heat flux τ are constant, it is used, for example, to describe the heat transfer in hereditary-elastic materials [4] and in attenuated gases [5-8]. In [9], it was shown that experimental data on the propagation of heat pulses in solids at low temperature are in good agreement with calculations using Eq. (2). It was noted in [10] that "the phenomenon of second sound observed long ago in liquid helium arose specifically when using a heat-conduction equation of hyperbolic type." The use of Eq. (2) in thermoelasticity problems was considered in [11], which also has an additional bibliography on the given problem.

Under certain assumptions, Eq. (2) may be obtained from the Boltzmann equation using the 13-moment method [12, 13]. A nonrigorous but clear derivation of Eq. (2) is possible on the basis of the simplest molecular-kinetic considerations [8], if the lag is taken into account in the simplest manner. The heat flux transferred by particles through an isolated area is not produced immediately at the instant when the temperature gradient acts but some time later, when the particles actually reach the area. In the first approximation

$$-\varkappa \left.\frac{\partial T}{\partial x}\right|_t = W|_{t+\tau} = \left.\left(W + \tau \frac{\partial W}{\partial t}\right)\right|_t$$

and hence Eq. (2) is obtained. Rewriting Eq. (2) in the form

$$\frac{\partial W}{\partial t} = -\frac{W - W_F}{\tau},$$

it may be regarded as the equation for relaxation of the heat flux to its quasi-equilibrium value determined by the Fourier law.

Recently, interest in Eq. (2) has grown because of the proposal that it be used to describe the electronic heat conduction of hot plasma [13-16]. It must be emphasized that Eq. (2) becomes nonlinear here, since τ and $\varkappa$ depend on the state of the medium. For example, for completely ionized plasma, a possible simplication is $\varkappa \sim T^{5/2}, \tau \sim T^{3/2}$ [13-17].

Below, the properties of the mathematical model of heat transfer based on Eq. (2) are investigated; heat transfer in a motionless homogeneous (it is assumed that $\rho \equiv 1$, C_V = const) medium described by the following system of equations is investigated

$$C_V \frac{\partial T}{\partial t} + \frac{\partial W}{\partial x} = 0, \tag{3}$$

$$\tau \frac{\partial W}{\partial t} + \varkappa \frac{\partial T}{\partial x} + W = 0. \tag{4}$$

3°. When $\varkappa$ = const, τ = const, Eqs. (3) and (4) reduce to the equation

$$\tau \frac{\partial^2 T}{\partial t^2} + \frac{\partial T}{\partial t} - \frac{\varkappa}{C_V} \frac{\partial^2 T}{\partial x^2} = 0, \tag{5}$$

which is known to be a particular case of the so-called telegraph equation and is called the hyperbolic heat-conduction equation [12, 18]. With an arbitrary temperature dependence of $\varkappa$ and τ, it is necessary to solve Eqs. (3) and (4), which form a system of hyperbolic type. The corresponding characteristic equation takes the form

$$\frac{dx}{dt} = \pm c_1, \quad c_1 = \sqrt{\varkappa / (C_V \tau)}. \tag{6}$$

The quantity c_1 is understood to be the rate of heat propagation [5, 6, 12, 18]. If $\varkappa = \varkappa_0 T^{a}$, $\tau = \tau_0 T^{a_1}$, then $c_1 = \sqrt{\varkappa_0 / (C_V \tau_0)}\, T^{(a-a_1)/2}$, and, in view of the temperature dependence of the local slope of the characteristics, characteristics of a single family may intersect. Hence, heat transfer may be accompanied by a phenomenon of shock-wave type in gas dynamics, i.e., discontinuities of the temperature and heat flux.

4°. Some solutions of Eqs. (3) and (4) with constant $\varkappa$ and τ are now given. There have been numerous investigations of Eqs. (3) and (4) in this case; see, for example, [4-8, 10-12, 18-20] and the literature cited there. To solve linear Eq. (5), to which the given system reduces, the classical methods of mathematical-physics equations may be used [21]; sometimes, they may even be solved accurately.

a) The solution of the problem of an instantaneous plane heat source (at time t = 0 in plane x = 0 per unit area, the energy liberated is $E_0 = C_V Q_0$; T(x, 0) = 0 when x ≠ 0) takes the form [8]

$$T(x, t) = Q_0 \sqrt{\frac{C_V}{2\varkappa\tau}} \exp\left(-\frac{t}{2\tau}\right) I_0\left(\sqrt{\frac{t^2}{\tau^2} - \frac{C_V}{\varkappa\tau} x^2}\right) H(c_1 t - x), \tag{7}$$

where $H(z)$ is a unit Heaviside step function: $H(z) = 0$ when $z < 0$ and $H(z) = 1$ when $z \geqslant 0$; I_0 is a modified zero-order Bessel function.

In contrast to the solution of the analogous problem for the ordinary heat-conduction equation ($\tau = 0$), which takes the form [21]

$$T(x, t) = Q_0 \sqrt{\frac{C_V}{4\pi\varkappa t}} \exp\left(-\frac{C_V x^2}{4\varkappa t}\right), \tag{8}$$

the temperature is zero when $x > c_1 t$ according to Eq. (7); heat propagates at finite velocity c_1, and at the thermal wavefront there is a temperature discontinuity. At small times, the behavior of the solution of the ordinary and hyperbolic heat-conduction equations — Eqs. (8) and (7) respectively — differs most significantly. At the same time, as $t \to \infty$, the asymptotes of Eqs. (7) and (8) coincide, as may be readily established using the asymptotic representation of the function I_0, valid at large values of the argument

$$I_0(z) = \frac{1}{2\pi z} e^z (1 + O(1/z)).$$

b) Consider the evolution of an initial temperature and heat-flux distribution in a heat-insulated region of finite size. Suppose that, on the segment $0 \leq x \leq l$, heat propagation is described by Eqs. (3) and (4); the initial conditions are given in the form

$$T(x, 0) = T_0(x);\ W(x, 0) = W_0(x);\ 0 \leqslant x \leqslant l, \tag{9}$$

and the boundary conditions are $W(0, t) = W(l, t) = 0$. The solution of the problem obtained by the variable-separation method is written as follows

$$T(x, t) = a_0 + \sum_{n=1}^{\infty} a_n^+ \exp(S_n^+ t) \cos\frac{\pi n x}{l} + \sum_{n=1}^{\infty} a_n^- \exp(S_n^- t) \cos\frac{\pi n x}{l},$$

$$W(x, t) = \sum_{n=1}^{\infty} b_n^+ \exp(S_n^+ t) \sin\frac{\pi n x}{l} + \sum_{n=1}^{\infty} b_n^- \exp(S_n^- t) \sin\frac{\pi n x}{l},$$

where

$$S_n^{\pm} = -\frac{1}{2\tau} \pm \frac{1}{2\tau} \sqrt{1 - 4\tau\pi^2\varkappa n^2/(l^2 C_V)},$$

and the series coefficients $a_n^{\pm}$ and $b_n^{\pm}$ are uniquely determined by the initial data of Eq. (9).

Some features of the given solution may be noted.

1) As $\tau \to 0$, $S_n^- \to -\infty$, and S_n^+ tends to the corresponding damping decrement for the parabolic equation; $S_n^+ \to -\lambda_n$, $\lambda_n = \varkappa^2\pi^2 n^2/(l^2 C_V)$; see [21], for example.

2) For any set of values $\varkappa$, τ, C_V, l, a number N is found such that any harmonic $n \geqslant N$ will undergo oscillations in time, $\mathrm{Im}(S_n^{\pm}) \neq 0$. In themselves, these harmonics will be damped in proportion to $\exp(-t/2\tau)$ since $\mathrm{Re}(S_n^{\pm}) = -1/(2\tau)$ in this case.

3) At specified $\varkappa$, C_V, l, there is a "critical" value $\tau = \tau_*$. When $\tau > \tau_*$, all the harmonics, beginning with the first, undergo oscillations in time. The "critical" value of the heat-flux relaxation time is

$$\tau_* = C_V l^2 / (4\pi^2\varkappa)$$

and decreases together with the length of the region l. When $\tau > \tau_*$ the solution is a superposition of oscillations damping in proportion to $\exp(-t/2\tau)$.

4) If $\tau < \tau_*$, then at least the first harmonic does not undergo oscillations over time. The most slowly damping harmonic in this case is that which corresponds to a plus sign of the square root when n = 1. When $t \to \infty$, the solution tends to regular conditions

$$W(x, t) \simeq \bar{W}_0 \exp\left[\left(-\frac{1}{2\tau} + \frac{1}{2\tau}\sqrt{1 - 4\tau\pi^2\varkappa/(l^2 C_V)}\right)t\right]\sin(\pi x/l),$$

$$T(x, t) \simeq \frac{1}{l}\int_0^l T_0(x)\,dx + \frac{l}{2\pi\varkappa}\bar{W}_0\left(1 + \sqrt{1 - \frac{4\tau\pi^2\varkappa}{l^2 C_V}}\right) \times$$

$$\times \cos\frac{\pi x}{l}\exp\left[\left(-\frac{1}{2\tau} + \frac{1}{2\tau}\sqrt{1 - \frac{4\tau\pi^2\varkappa}{l^2 C_V}}\right)t\right].$$

5) The physical meaning of the inequality

$$\tau < \tau_* = \frac{C_V l^2}{4\pi^2\varkappa} \qquad (10)$$

may now be elucidated. Taking account of Eq. (6), Eq. (10) may be written in the following form for the rate of heat propagation c_1

$$c_1\tau < l/(2\pi). \qquad (11)$$

Since the relaxation time τ is of the order of the free-path time of the particles in the material, and the rate of heat propagation c_1 is of the order of the thermal velocity, the quantity on the left-hand side of Eq. (11) is of the order of the free-path length L. Hence, the meaning of Eq. (10) is that $L \lesssim l/2\pi$.

5°. Generally speaking, Eq. (3) and (4) admit of the existence of solutions where sign W = sign($\partial T/\partial x$) and the heat flux is directed toward increase in temperature. It is found that taking account of heat-flux relaxation leads to change in the basic thermodynamic inequality for irreversible processes regarding the production of entropy σ:

$$\sigma = \mathbf{W}\,\mathrm{grad}\,(1/T) \geqslant 0. \qquad (12)$$

In [22], Eq. (12) was given in more general form

$$\sigma = \left(\mathbf{W} + \tau\frac{\partial\mathbf{W}}{\partial t}\right)\mathrm{grad}\,(1/T) \geqslant 0.$$

Therefore, in describing heat transfer, using Eqs. (3) and (4) there is no violation of the second principle of thermodynamics, as may appear to occur on first glance.

6°. Now, letting $\varkappa = \varkappa_0 T^a$, $\tau = \tau_0 T^{a_1}$, C_V = const, the solution of the already-quasilinear system in Eqs. (3) and (4) is sought in the form of a traveling wave propagating against a constant background. The constant background assumed must obviously be characterized by some value $T = T_0$ and a zero value of W.

Introducing the independent variable $\xi = x - Dt$, where D = const is the velocity of the traveling wave, it is found that the desired solution must satisfy a system of ordinary differential equations

$$-C_V D\frac{\partial T}{d\xi} + \frac{dW}{d\xi} = 0, \qquad (13)$$

$$-D\tau_0 T^{a_1}\frac{dW}{d\xi} + \varkappa_0 T^a\frac{dT}{d\xi} + W = 0. \qquad (14)$$

Taking account of the values in the background, the integral of Eq. (13) is written in the form

$$W = C_V D (T - T_0). \tag{15}$$

Substituting Eq. (15) into Eq. (14), the system is reduced to a single ordinary differential equation

$$\frac{dT}{d\xi} \frac{C_V D^2 \tau_0 T^{a_1} - \varkappa_0 T^a}{C_V D (T - T_0)} = 1. \tag{16}$$

Without investigating all the possible parameter values in detail, consider the case corresponding to completely ionized plasma ($a = 5/2$, $a_1 = 3/2$). The solution of Eq. (16) then takes the form

$$\xi - \xi_0 = -\frac{2}{5} \frac{\varkappa_0}{C_V D} T^{5/2} + \frac{\varkappa_0}{C_V D} \left(\frac{C_V D^2 \tau_0}{\varkappa_0} - T_0 \right) \times \left[\frac{2}{3} T^{3/2} + 2T_0 T^{1/2} + T_0^{3/2} \ln \left| \frac{\sqrt{T} - \sqrt{T_0}}{\sqrt{T} + \sqrt{T_0}} \right| \right]. \tag{17}$$

The appearance of the free parameter ξ_0 is due to invariance of Eqs. (13) and (14) relative to the transformation $\xi' = \xi - \xi_0$. The arbitrariness associated with the choice of ξ_0 may be eliminated by specifying the initial (when $t = 0$) position of the desired traveling wave.

7°. The qualitative character of Eq. (17) is determined by the relation between T_0 and the value of the combination of determining parameters $T_{00} = C_V D^2 \tau_0 / \varkappa_0$ or, in other words, the relation between the traveling-wave velocity D and the velocity of heat propagation against the background $c_1|_{T=T_0} = \sqrt{\varkappa_0 T_0 / C_V \tau_0}$. Integral curves are shown in Fig. 1 for the cases: a) $T_0 = 0$ ($c_1 = 0$); b) $0 < T_0 < T_{00}$ ($0 < c_1 < D$); c) $T_0 = T_{00}$ ($c_1 = D$); d) $T_0 > T_{00}$ ($c_1 > D$). The corresponding family is obtained by shifting the given curves parallel to the axis 0ξ.

Considering $\xi = x - Dt$, it is readily evident that some physical process over time (t increases) may correspond to points of the integral curves traced with decrease in ξ. In the general case, continuous transition from the background to the integral curve is impossible, and the permissible solutions of traveling-wave type must be sought in the class of discontinuous functions composed of individual sections of integral curves with a discontinuous transition to the background.

8°. To obtain the conditions which must be satisfied at the discontinuity of the desired solution, Eqs. (3) and (4) are written in conservative [23] form

$$\frac{\partial T}{\partial t} + \frac{\partial}{\partial x} \left(\frac{W}{C_V} \right) = 0, \quad \frac{\partial W}{\partial t} + \frac{\partial}{\partial x} \left(\frac{\varkappa_0}{\tau_0} \frac{T^{a - a_1 + 1}}{a - a_1 + 1} \right) = -\frac{W}{\tau_0 T^{a_1}}.$$

Integration of the equations obtained in the small, compressing to zero volume, region of variation of the independent variables (x, t) which includes the discontinuity line gives the following relations as analogs of the Hugoniot conditions when $a = 5/2$, $a_1 = 3/2$

$$W_* = C_V D (T_* - T_0), \tag{18}$$

$$T_* = 2 C_V D^2 \tau_0 / \varkappa_0 - T_0 \text{ or } T_* - T_0 = 2 (C_V D^2 \tau_0 / \varkappa_0 - T_0). \tag{19}$$

At arbitrary a and a_1, Eq. (19) is replaced by the condition

$$T_*^{a - a_1 + 1} - T_0^{a - a_1 + 1} = \tau_0 C_V D^2 (a - a_1 + 1) \varkappa_0^{-1} (T_* - T_0).$$

Here D is the rate of propagation of the discontinuity. For solutions of traveling-wave type, of course, it coincides with the velocity of wave motion, and therefore the same notation may be used; T_0 is the temperature value to the right of the discontinuity, the temperature of the unperturbed background; an asterisk denotes values following the discontinuity, i.e., to the left of it.

In connection with the foregoing, it must be noted that in [24], where Eqs. (3) and (4) were also investigated for parameter values corresponding to the electronic heat conduction of hot plasma, errors were introduced in finding self-similar solutions depending on a single variable of the form $\zeta = x/(1 - \alpha t)$. In the solutions obtained there, only the temperature undergoes a discontinuity, while the heat flux remains constant at the thermal wavefront.

As is evident from Fig. 1, Eq. (19) uniquely determines the temperature behind the discontinuity and hence also the position of the discontinuity in terms of the specified parameters.

Note that, when $c_1 < D$, or $T_0 > T_{00} = C_V D^2 \tau_0/\varkappa_0$ (Fig. 1d), Eq. (19) either leads the temperature in the region of negative values or places the value of T_* on the lowest branch $T < T_{00}$ of the integral curve, which is associated with the metastable solution. In this case, no solution of the problem exists in the given class of functions (with a discontinuity) but there is a continuous solution tending asymptotically to the background as $\xi \to \infty$, shown by the upper curve lying entirely in the region $T > T_0$ in Fig. 1d.

When $c_1 \geqslant D$ or $T_0 \leqslant T_{00}$, the solution is determined uniquely, as already noted; when $c_1 \to D$ ($T_0 \to T_{00}$), the discontinuous solution transforms to a continuous solution, as follows from Eqs. (18) and (19).

Analysis of the behavior of the characteristics in the vicinity of the discontinuity, as well as direct numerical computer calculations of Eqs. (3) and (4) with the corresponding initial and boundary conditions, shows the evolutionary character of the solution obtained.

NOTATION

x, spatial coordinate; t, time; T, temperature; W, heat flux; W_F, heat flux calculated from the Fourier law; $\varkappa$, thermal conductivity; τ, relaxation time of heat flux; ρ, density; C_V, specific heat; c_1, characteristic rate of heat propagation; D, velocity of motion of traveling wave.

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